Design and Analysis of Conveyor Pulleys Using a Precise Finite-Element Model based on the Hamiltonian Form of Elasticity

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Why a pulley program?

* A lot, almost all, pulleys designed today use tools that are antiquated. There are a few exceptions – some FEA – PPI.
* The better tools are based on Lange and Schmoltzi (1960's and 1970's) technology.
* Many use models from Sitzwhol (1948) (also know as the Link Belt Method) and simple plate bending equations from Roark (1950's)
* I enjoy Numerical Modeling and Computers. This is a challenging exercise. – Covey Habit 7: Sharpen the Saw

Precise Finite-Element Model based on the Hamiltonian Form of Elasticity

* Xingjun Qui & Vinit Sethi, "A new Pulley Stress Analysis Method Based on Modified Transfer Matrix", Bulk Solids Handling, Nov. 1993.
* Xingjun Qui, et. al., "Precise Finite-Element Model for Pulleys Based on the Hamiltonian Form of Elasticity", American Society of Civil Engineers, 2014.

Major Improvement in the New Work

"In Qui and Sethi's (1993) work, the major approximate solution employed was Timoshenko and Woinowsky-Krieger's (1959) approximate solution to the cylindrical shell, in which the terms of circumferential displacement in strain-displacement relations are neglected for mathematical convenience. The consequence of this approximation in that four of six modes of rigid body motions are no longer preserved in the solution and errors are unavoidable in certain cases."

Math for the Shell – a lot more!

Modified Transfer Matrix

1993

$$J_m = \begin{bmatrix} 0 & -\frac{\mu m}{R} & -\frac{\mu}{R} & 0 & \frac{1-\mu^2}{2\pi EtR} & 0 & 0 & 0 \\ \frac{m}{R} & 0 & 0 & 0 & 0 & \frac{1+\mu}{\pi EtR} & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\mu m^2}{R^2} & 0 & 0 & 0 & 0 & \frac{6(1-\mu^2)}{\pi Et^3R} \\ 0 & 0 & 0 & 0 & 0 & -\frac{m}{R} & 0 & 0 \\ 0 & \frac{2\pi Etm^2}{R} & \frac{2\pi Etm}{R} & 0 & \frac{\mu m}{R} & 0 & 0 & 0 \\ 0 & \frac{2\pi Etm}{R} & \frac{2\pi Et}{R}\left(1+\frac{t^2m^4}{12R^2}\right) & 0 & \frac{\mu}{R} & 0 & 0 & \frac{\mu m^2}{R^2} \\ 0 & 0 & 0 & \frac{\pi Et^3m^2}{3(1+\mu)R} & 0 & 0 & 1 & 0 \end{bmatrix} \tag{C30}$$

Precise Finite-Element Model

2014

$$A_n = \begin{bmatrix} 0 & -\frac{\mu n}{R} & -\frac{\mu}{R} & 0 \\ \frac{EtnR}{EtR^2+4(1-\mu^2)D} & 0 & 0 & \frac{4(1-\mu^2)nD}{EtR^2+4(1-\mu^2)D} \\ 0 & 0 & 0 & -1 \\ 0 & -\frac{\mu n}{R^2} & -\frac{\mu n^2}{R^2} & 0 \end{bmatrix} \tag{47}$$

$$B_n = \begin{bmatrix} \frac{1-\mu^2}{2\pi REt} & 0 & 0 & 0 \\ 0 & \frac{(1+\mu)R}{\pi EtR^2+4\pi(1-\mu^2)D} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2\pi RD} \end{bmatrix} \tag{48}$$

$$C_n = \begin{bmatrix} \frac{4\pi(1-\mu)n^2EtD}{EtR^3+4(1-\mu^2)RD} & 0 & 0 & -\frac{4\pi(1-\mu)n^2EtD}{EtR^2+4(1-\mu^2)D} \\ 0 & \frac{2\pi n^2Et}{R}+\frac{2\pi(1-\mu^2)n^2D}{R^3} & \frac{2\pi nEt}{R}+\frac{2\pi(1-\mu^2)n^3D}{R^3} & 0 \\ 0 & \frac{2\pi nEt}{R}+\frac{2\pi(1-\mu^2)n^3D}{R^3} & \frac{2\pi Et}{R}+\frac{2\pi(1-\mu^2)n^4D}{R^3} & 0 \\ -\frac{4\pi(1-\mu)n^2EtD}{EtR^2+4(1-\mu^2)D} & 0 & 0 & \frac{4\pi n^2(1-\mu)D}{R}-\frac{16\pi(1+\mu)(1-\mu^2)n^2D^2}{EtR^3+4(1-\mu^2)RD} \end{bmatrix} \tag{49}$$

$$D_n = \begin{bmatrix} 0 & -\frac{EtnR}{EtR^2+4(1-\mu^2)D} & 0 & 0 \\ \frac{\mu n}{R} & 0 & 0 & \frac{\mu n}{R^2} \\ \frac{\mu}{R} & 0 & 0 & \frac{\mu n^2}{R^2} \\ 0 & -\frac{4(1-\mu^2)nD}{EtR^2+4(1-\mu^2)D} & 1 & 0 \end{bmatrix} \tag{50}$$

where

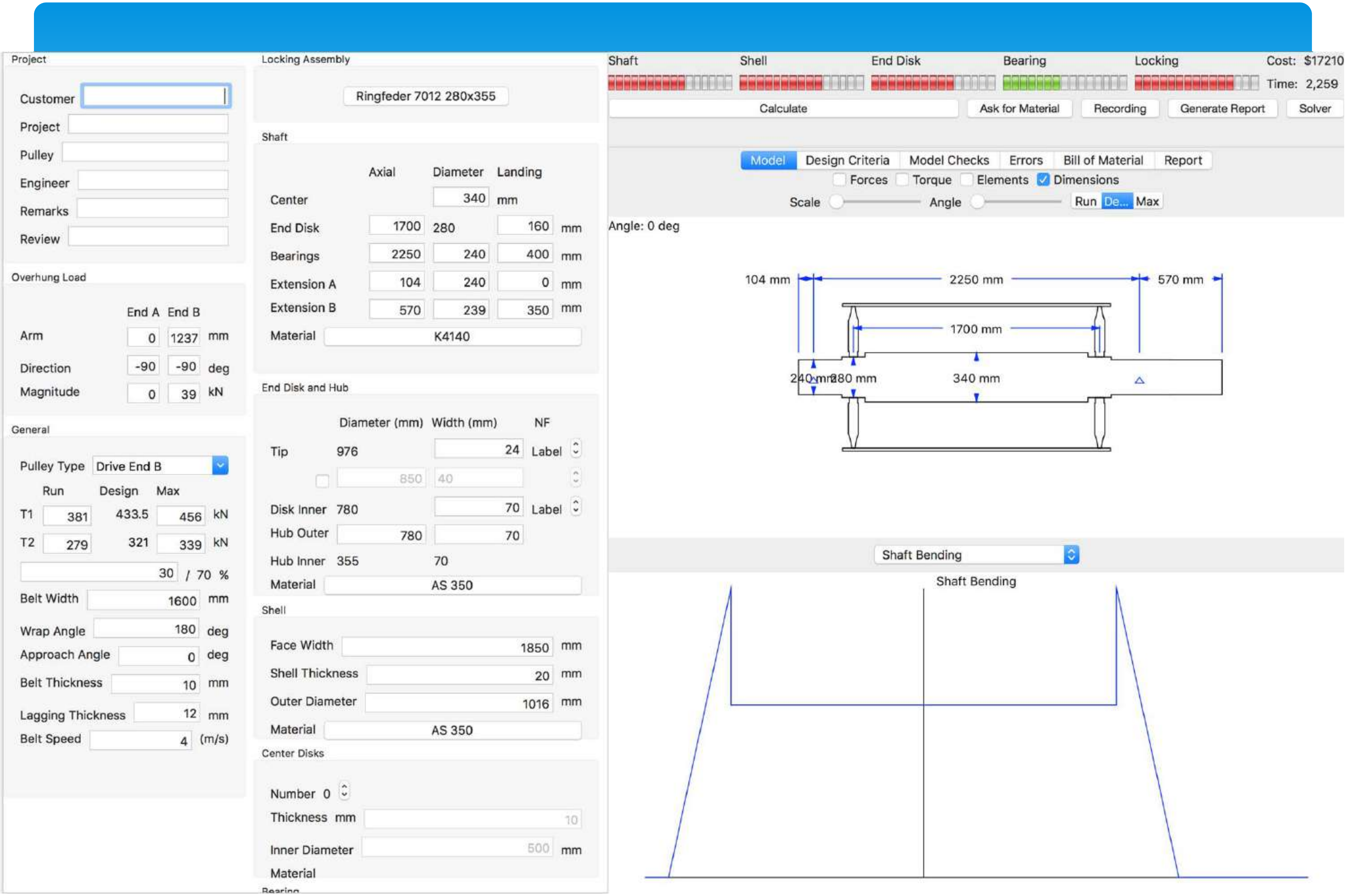
$$K_n = \begin{bmatrix} -T_{n,qp} & 0 \\ -T_{n,pp} & -I \end{bmatrix}^{-1} \begin{bmatrix} -T_{n,qq} & I \\ -T_{n,pq} & 0 \end{bmatrix}; \quad U_n = \begin{Bmatrix} q_n(0) \\ q_n(x) \end{Bmatrix};$$

$$F_{n,int} = \begin{Bmatrix} -p_n(0) \\ p_n(x) \end{Bmatrix}; \quad \text{and} \quad F_{n,ext} = \begin{bmatrix} -T_{n,qp} & 0 \\ -T_{n,pp} & -I \end{bmatrix}^{-1} \begin{Bmatrix} F_{n,q} \\ F_{n,p} \end{Bmatrix}$$

The following comments apply to these preceding equations:

1. The mathematical properties described by Eq. (51) guar-

Example Pulley



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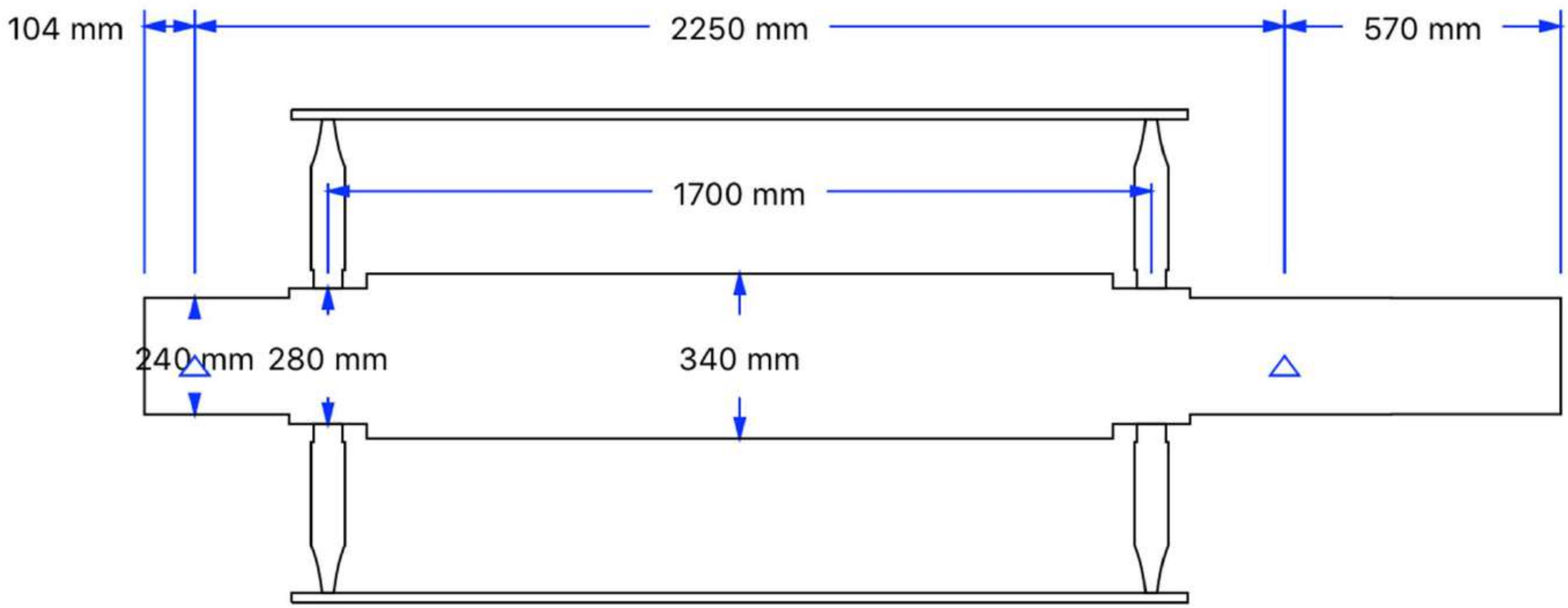
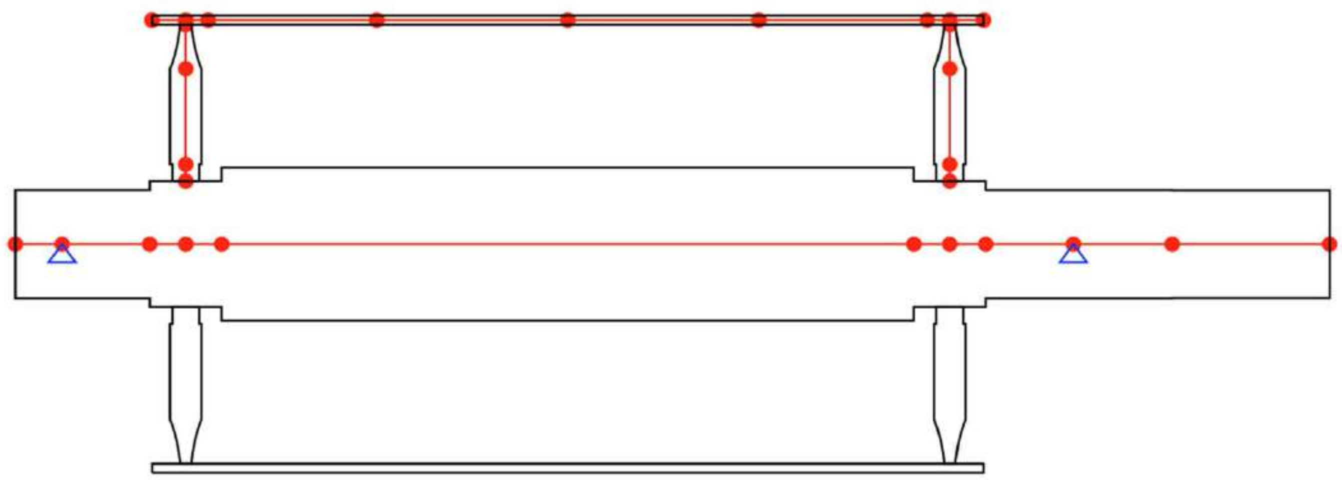
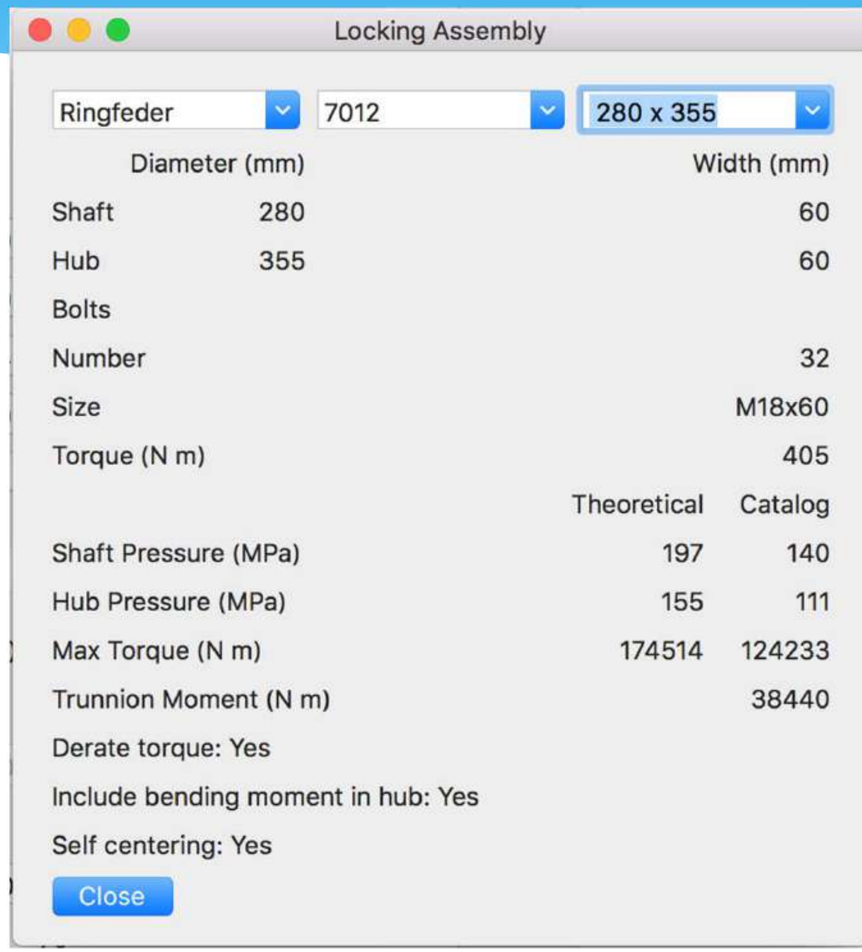
Shaft Dimensions



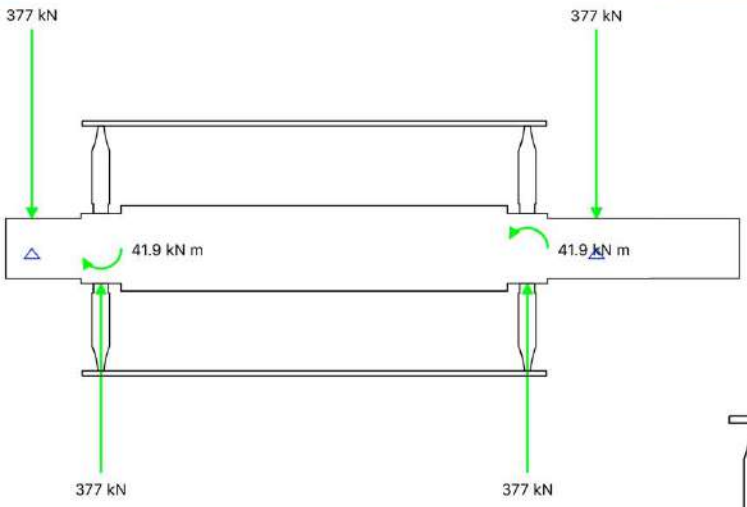
Finite Elements

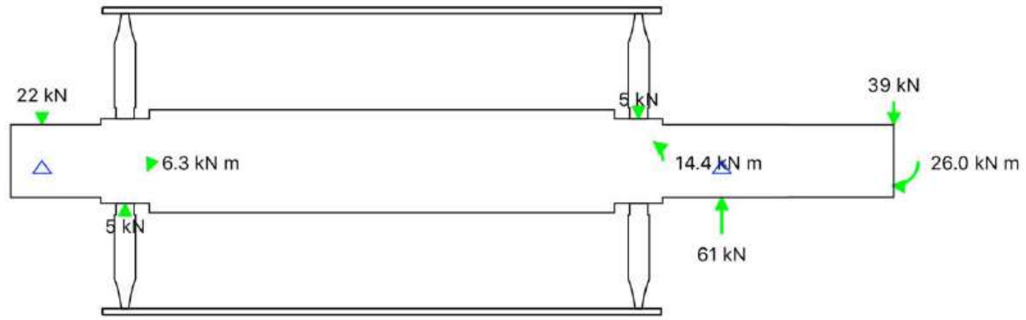


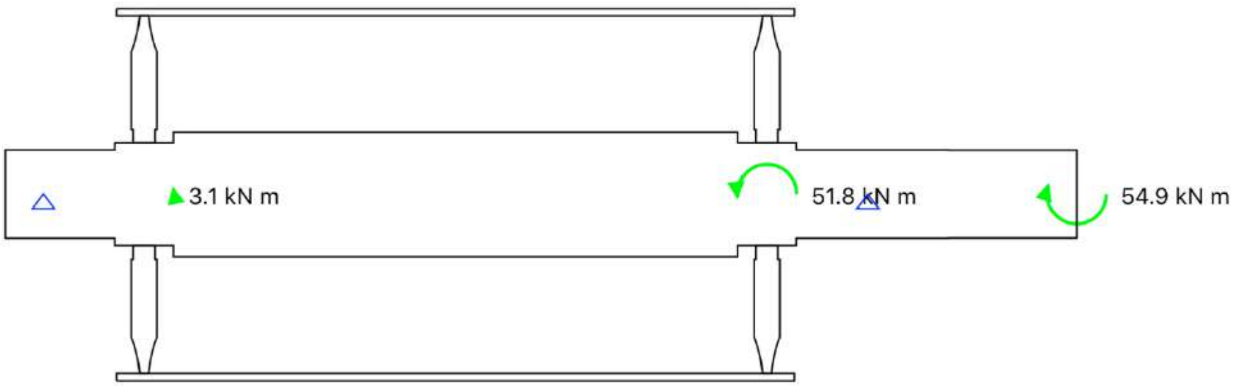
Locking Assembly



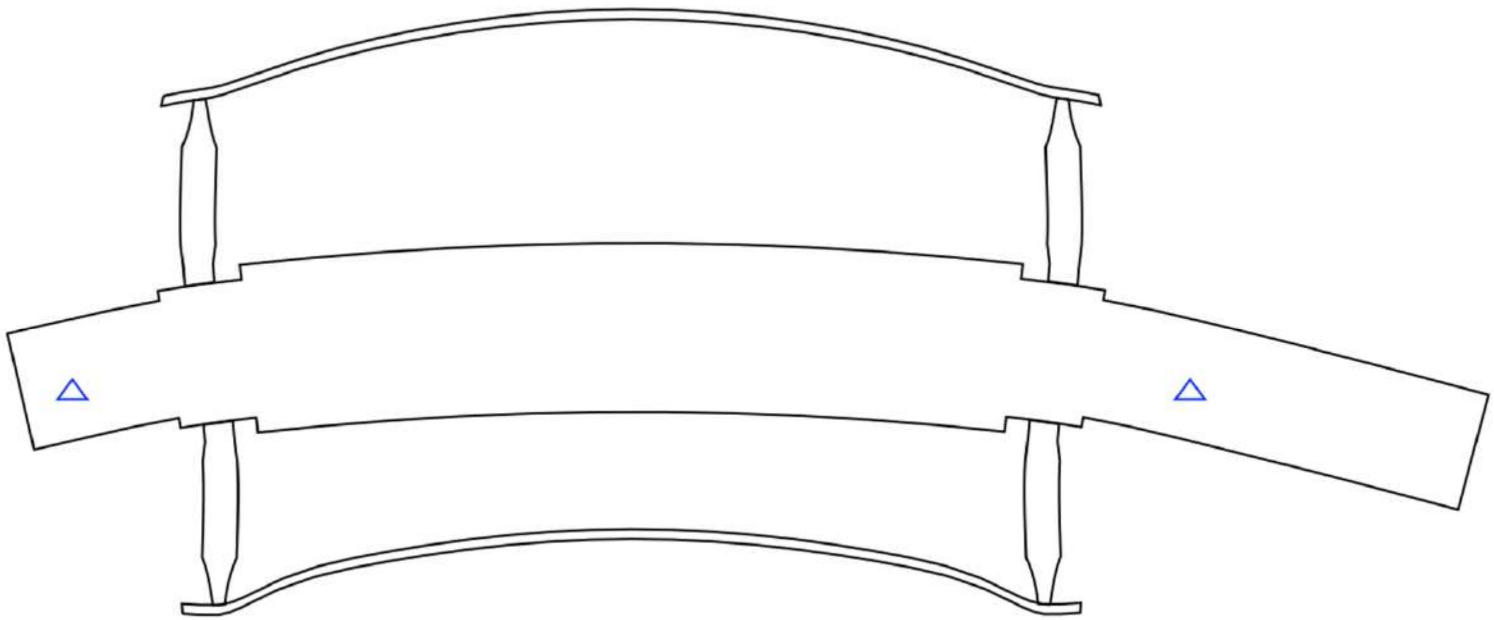
Free Body Diagrams of the Shaft



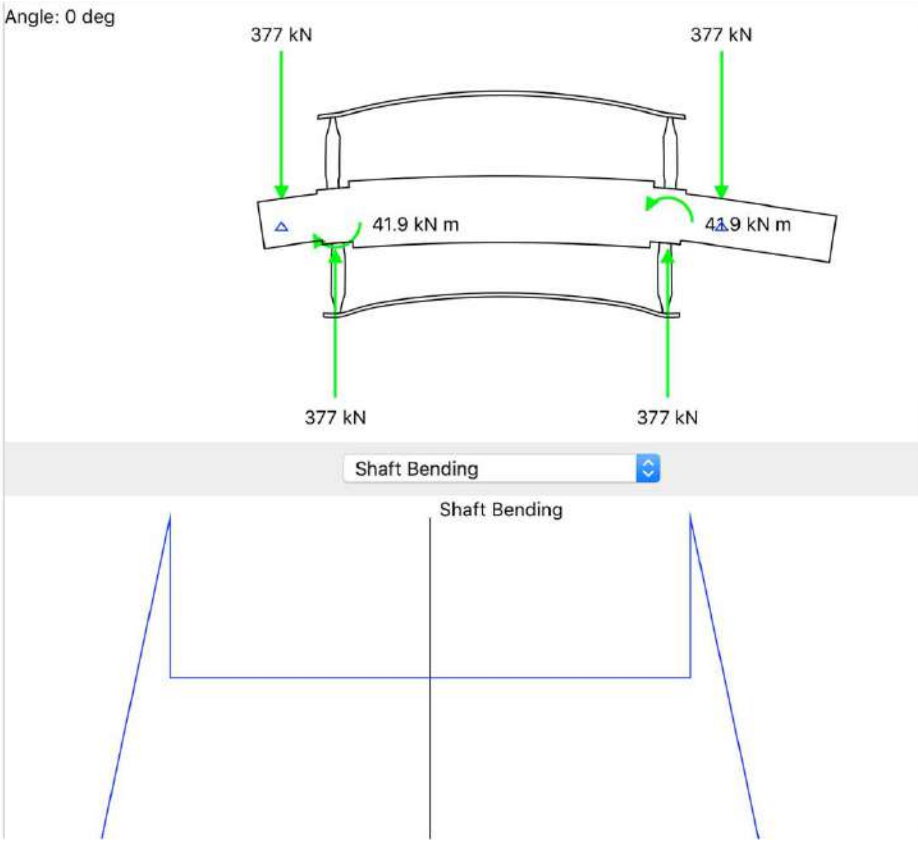


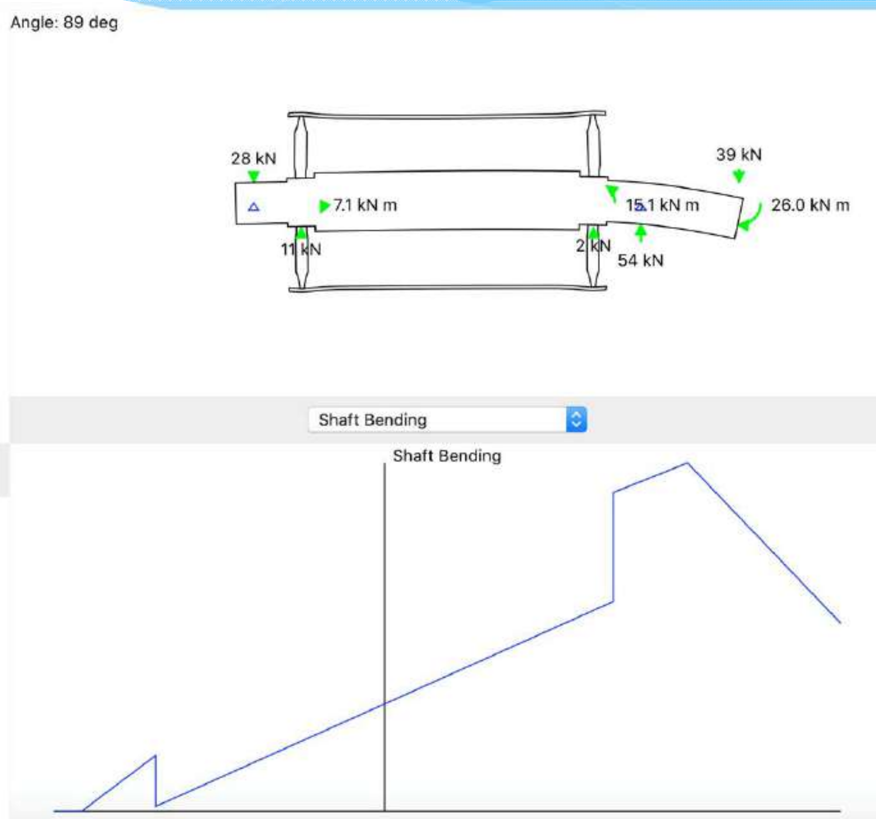


Displacements

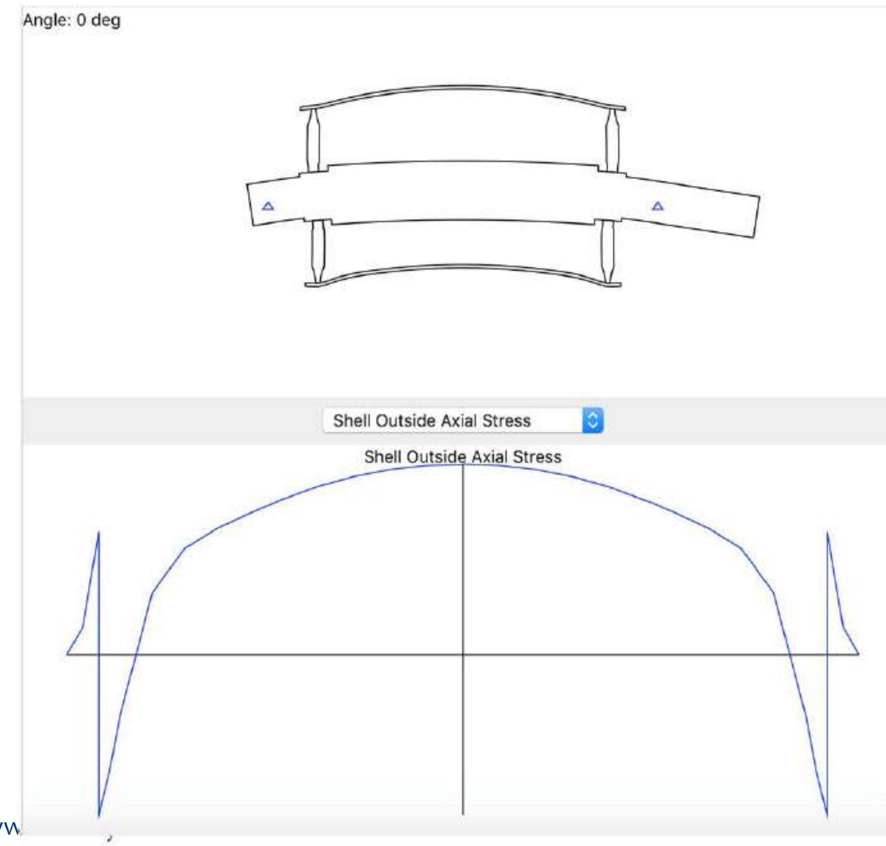


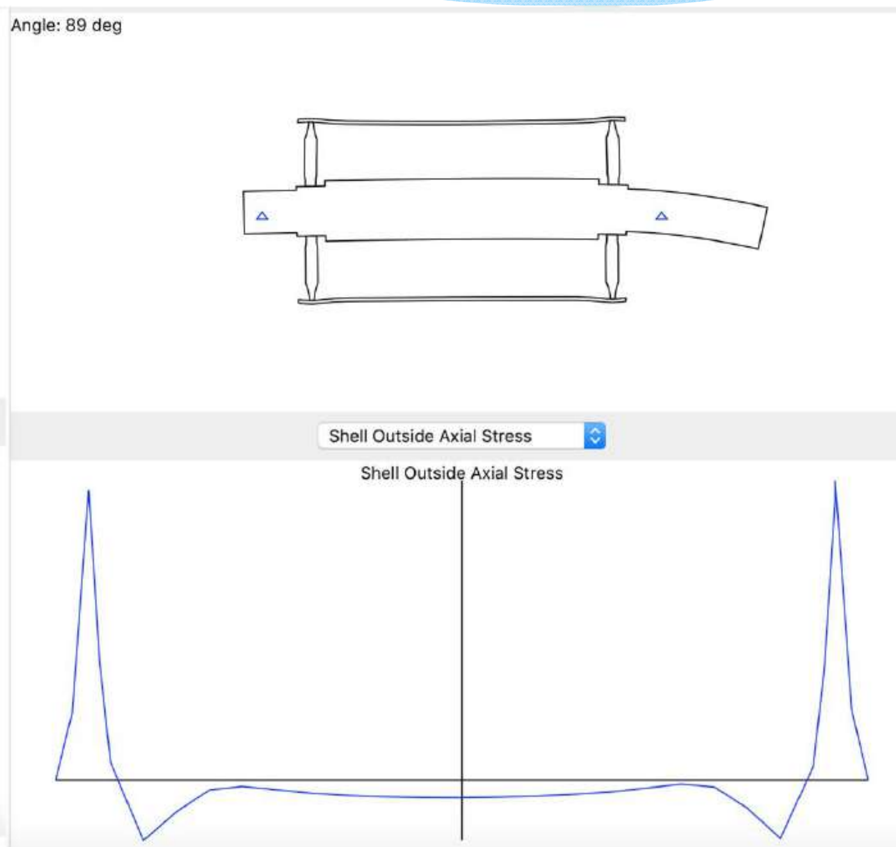
Shaft Moments



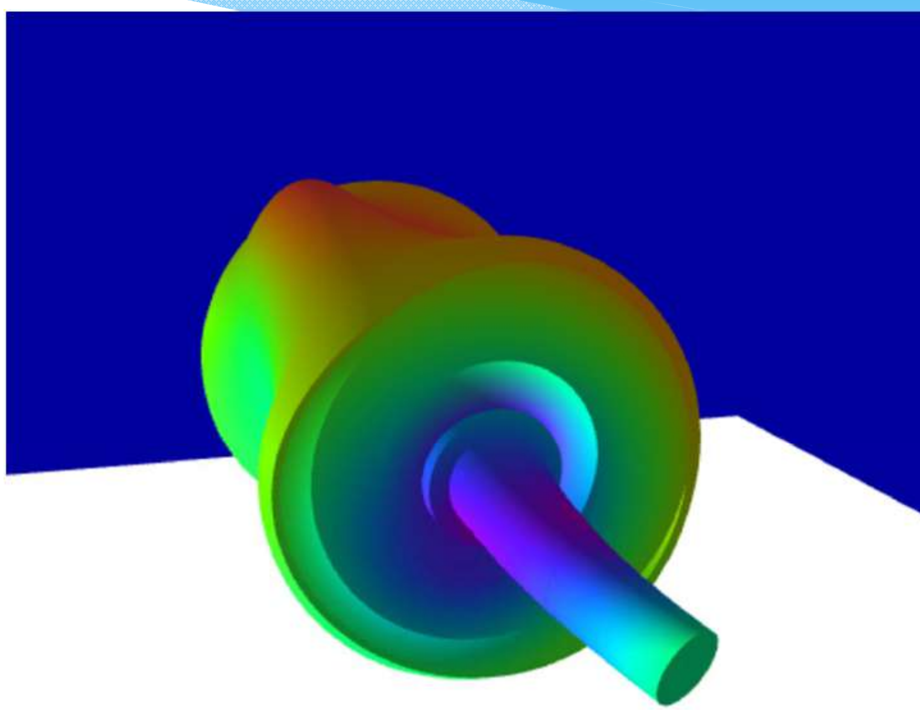


Shell Outer Stresses





3D Displacements



Checking The Model

Check	Location	A	B	Units	Difference	Limit	Status
Internal FEA							
Trunnion Moment	A	42.38	42.38	kN m	0	1	✓
Trunnion Moment	B	44.3	44.3	kN m	0	1	✓
Trunnion Moment Min/Max	A	42.38	42.38	kN m	0	1	✓
Trunnion Moment Min/Max	B	44.3	44.3	kN m	0	1	✓
Pull Force	A	377.3	377.3	kN	0	1	✓
Pull Force	B	377.3	377.3	kN	0	1	✓
Pull Force Min/Max	A	377.3	377.3	kN	0	1	✓
Pull Force Min/Max	B	377.3	377.3	kN	0	1	✓
Torque	A	3.06	3.06	kN m	0	1	✓
Torque	B	51.84	51.84	kN m	0	1	✓
Torque Min/Max	A	3.06	3.06	kN m	0	1	✓
Torque Min/Max	B	51.84	51.84	kN m	0	1	✓
Shaft Slope	A	0.000433	0.000433	Radians	0	1	✓
Shaft Slope	B	0.000439	0.000439	Radians	0	1	✓
Shaft Slope Horizontal	A	-8.166e-0	-8.166e-0	Radians	0	1	✓
Shaft Slope Horizontal	B	0.0001081	0.0001081	Radians	0	1	✓
Shaft Slope Vertical	A	-0.00042	-0.00042	Radians	0	1	✓
Shaft Slope Vertical	B	0.000425	0.000425	Radians	0	1	✓
FEA against Applied Load							
Hub Pressure	A	-95.14	-95.14	MPa	0	1	✓
Hub Pressure	B	-95.14	-95.14	MPa	0	1	✓
Total Torque		54.9	56.02	kN m	2.049	1	✗
Total Pull Force		754.5	754.5	kN	0	1	✓
Total Shaft Force		755.5	755.5	kN	0	1	✓
FEA against Analytical Approximations							
End Disk Stiffness	A	97.77	5044	MN m/rad	5059	1	✗
End Disk Stiffness	B	100.9	5044	MN m/rad	4902	1	✗

Design Checks

* We've always done it this way
* Results from simpler and incomplete models
* With modern computers and models what should we check?
* Which loads? Running? Maximum? or a "Design" Load?

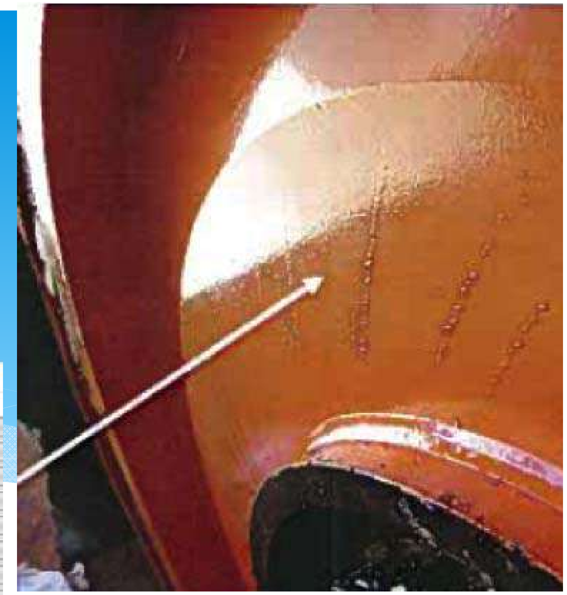
	Run	Design	Max	
T1	381	433.5	456	kN
T2	279	321	339	kN
	30		/ 70	%

Design Criteria

Criteria	Location	Load	Enforce	Limit	Actual	Units	Status	Level
Shell								
Fatigue Tangential Stress	Center	design	☑	85	62.41	MPa	✓	
Fatigue Max Von Mises Stress	Center	design	☑	55	56.15	MPa	✗	
Shaft								
AS1403 Safety Factor	OH A	max	☑	1.2	8.74e+		✓	
AS1403 Safety Factor	Bearing A	max	☑	1.2	7.43e+		✓	
AS1403 Safety Factor	Shoulder	max	☑	1.2	2.01		✓	
AS1403 Safety Factor	Edge End	max	☑	1.2	1.28		✓	
AS1403 Safety Factor	End Disk	max	☐	1.2	1.05		✗	
AS1403 Safety Factor	End Disk	max	☐	1.2	0.969		✗	
AS1403 Safety Factor	Edge End	max	☑	1.2	1.1		✗	
AS1403 Safety Factor	Shoulder	max	☑	1.2	1.62		✓	
AS1403 Safety Factor	Bearing B	max	☑	1.2	3.06		✓	
AS1403 Safety Factor	OH B	max	☑	1.2	1.44		✓	
ANSI Safety Factor	OH A	design	☑	1.5	1.16e+1		✓	
ANSI Safety Factor	Bearing A	design	☑	1.5	5.20e+		✓	
ANSI Safety Factor	Shoulder	design	☑	1.5	1.92		✓	
ANSI Safety Factor	Edge End	design	☑	1.5	1.8		✓	
ANSI Safety Factor	End Disk	design	☐	1.5	1.6		✓	
ANSI Safety Factor	End Disk	design	☐	1.5	1.48		✗	
ANSI Safety Factor	Edge End	design	☑	1.5	1.62		✓	
ANSI Safety Factor	Shoulder	design	☑	1.5	1.6		✓	
ANSI Safety Factor	Bearing B	design	☑	1.5	3.17		✓	
ANSI Safety Factor	OH B	design	☑	1.5	2.51		✓	

Design Criteria

Criteria	Location	Load	Enforce	Limit	Actual	Units	Status	Level
End Disk								
Radial Stress	End Disk	design	✓	55	55.5	MPa	✗	
Radial Stress	End Disk	design	✓	55	55.5	MPa	✗	
Radial Stress Excluding Hub Pressure	End Disk	design		55	56.7	MPa	✗	
Radial Stress Excluding Hub Pressure	End Disk	design		55	56.7	MPa	✗	
Hub Diameter	End Disk	design	✓	629	780	mm	✓	
Hub Diameter	End Disk	design	✓	636	780	mm	✓	
DIN 15018	A Inner	design	✓	1.10	0.26		✓	
DIN 15018	A Center	design	✓	1.10			n/a	
DIN 15018	A Outer	design	✓	1.10	0.84		✓	
DIN 15018	A Inner	design	✓	1.10	0.08		✓	
DIN 15018	A Center	design	✓	1.10			n/a	
DIN 15018	A Outer	design	✓	1.10	0.50		✓	
DIN 15018	B Inner	design	✓	1.10	0.06		✓	
DIN 15018	B Center	design	✓	1.10			n/a	
DIN 15018	B Outer	design	✓	1.10	0.50		✓	
DIN 15018	B Inner	design	✓	1.10	0.27		✓	
DIN 15018	B Center	design	✓	1.10			n/a	
DIN 15018	B Outer	design	✓	1.10	0.85		✓	
Locking Assembly								
Bending Moment	End Disk	design	✓	38440	42380	N-m	✗	
Bending Moment	End Disk	design	✓	38440	44300	N-m	✗	
Torque Factor	End Disk	run	✓	2	42.7		✓	
Torque Factor	End Disk	run	✓	2	2.51		✓	
Torque Factor	End Disk	max	✓	1.5	36.4		✓	
Torque Factor	End Disk	max	✓	1.5	2.14		✓	
Restrained Linear Deflection	Center	design		0.0005	0.0001	mm/	✓	
Free Angular Deflection	End Disk	design		0.0145	0.0006	radians	✓	
Free Linear Deflection	Center	design		0.0005	0.0001	mm/	✓	
Restrained Angular Deflection	End Disk	design		0.0145	0.0004	radians	✓	
Restrained Angular Deflection	End Disk	design		0.0145	0.0004	radians	✓	

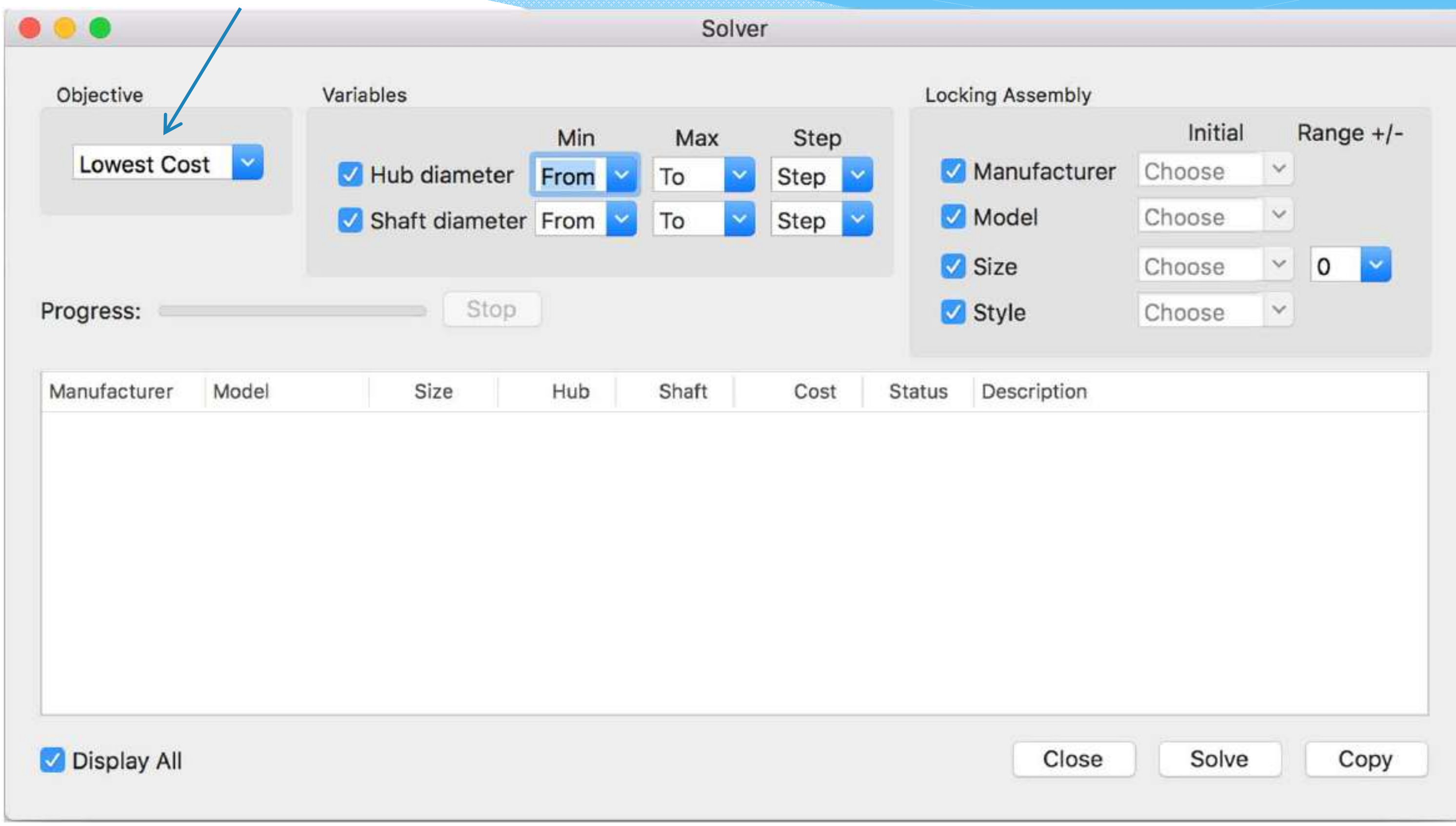


Next Steps – Bill of Material and Operations

Item	Description	Quantity	Units	Cost	Extended
▼ Pulley	Pulley	1	Each	1.7e+04	1.7e+04
▼ EndDiskB	994.5 x 355 x 70 Plate AS 350 70mm	1	Each	1.4e+03	1.4e+03
PM00000115	Plate AS 350 70mm	575.9	Kg	2.4	1.4e+03
PM00000021	Oxy Cutter	0.5578	Hr	1	0.56
▼ PM00002024	Ring Backing 270 x 390 x 16	1	Each	40	40
PM00000020	Plasma Cutter	0.2141	Hr	1	0.21
PM00000133	Plate 1045 16mm	17.25	Kg	2.3	40
▶ EndDiskA	994.5 x 355 x 70 Plate AS 350 70mm	1	Each	1.4e+03	1.4e+03
PM00001277	Ringfeder 7012 280x355	2	Each	7e+02	1.4e+03
▼ Shaft	340 Finished Dia. x 2932 Bar 4140 360mm	1	Each	9.8e+03	9.8e+03
PM00000017	Normalize	2.395	Ton	3.2e+02	7.7e+02
PM00000014	Bar Ultrasound	1	Each	90	90
PM00000252	Bar 4140 360mm	2395	Kg	3.6	8.6e+03
PM00000015	Rough Machine	0.5542	Hr	90	50
PM00000016	Fine Machine	3.531	Hr	90	3.2e+02
▼ Shell	994.5 ID x 1856 Plate AS 350 28mm	1	Each	3.1e+03	3.1e+03
PM00000019	Weld MIG	19.66	Kg	1.2	24
PM00000012	Weld MIG	0.8932	Hr	90	80
PM00000013	Weld Prep	1.531	Hr	90	1.4e+02
PM00000107	Plate AS 350 28mm	1293	Kg	2.2	2.8e+03
PM00000010	Plate Roll	0.5399	Hr	90	49

Next Steps – Solver & Design Optimization

Objective: Lowest Cost, Lightest Weight, Standardized



Next Steps – Standardize and Rationalize

* Rationalization of Pulley Sizes and Pulley Designs for Sizes Larger that “Cema Class” or “Mine Duty”
* Deeper investigation into Locking Assemblies and their Bending Capacity
* Optimized, standard designs, for end disk.

Why not just use a commercial FEA package like Ansys?

* It's REALLY hard to get right. You need to have very advanced FEA skills.
* Once you get the results you have stresses and strains at a bunch of points but now need to consider fatigue as it rotates and apply design standards.

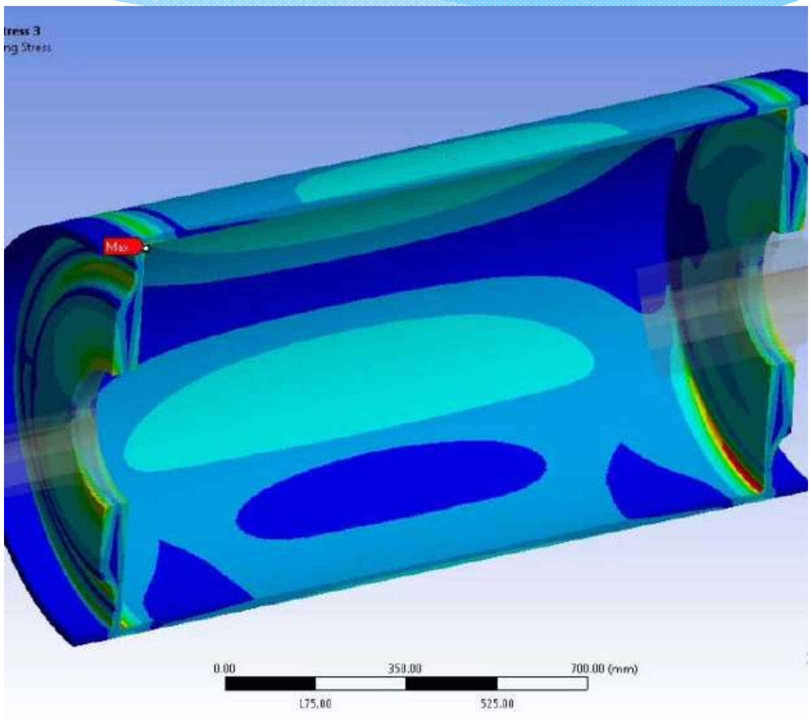


Image from Lorbrand website.

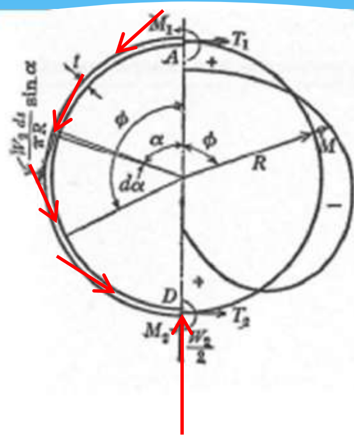
Conclusions

* The pressure from modern locking assemblies influences the stresses in the pulley shell.
* Locking assembly manufactures don't calculate the hub and shaft pressures the same way. Some are not conservative!
* The end disk equations of Lange and Schmoltzi don't handle, high tension, small diameter pulley very well.
* Many "traditional" design criteria need to be forgotten. In many cases they limit modern design options.

Outline

- Shell Calculations
  - Sitzwohl (1948)
  - Lange (1963)
  - Xiangjun Qui
- End Disk
  - Williams (1950)
  - Lange (1963)
  - Schmoltzi
  - Xiangjun Qui
- Shaft
- 3D FEA

Shell - Sitzwohl

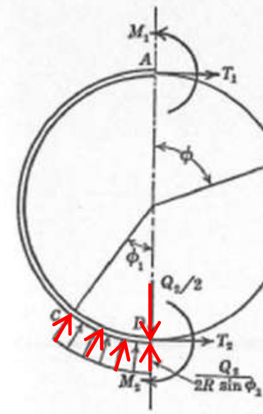


Case II. Shell, of length dx, loaded with shearing forces acting parallel to the circumference.

$$M = \frac{W_2R}{2\pi}\left(1 - \phi \sin \phi - \frac{\cos \phi}{2}\right).$$

$$T = \frac{W_2}{2\pi}\left(\frac{3}{2}\cos \phi - \phi \sin \phi\right).$$

For derivation of equations se



Case VII. Symmetrical uniform radial support.

$$M\Big|_A^C = \frac{Q_2R}{2\pi \sin \phi_1}\left[\phi_1 - \sin \phi_1 + (\phi_1 \cos \phi_1 - \sin \phi_1) \cos \phi\right].$$

$$M\Big|_C^D = \frac{Q_2R}{2\pi \sin \phi_1}\left[\phi_1 - \pi - \sin \phi_1 + (\phi_1 - \pi) \cos \phi_1 \cos \phi + \pi \sin \phi_1 \sin \phi - \sin \phi_1 \cos \phi\right].$$

* Van den Broek, 1942, Chapter on Redundant Beams, Pipe Problems, Case II and Case VII.

Shell - Sitzwohl

* Sitzwohl, Josef, Welded Conveyor Pulleys – Analysis of Stresses in Pulley Shell, 1948, Australia. (Available on Amazon)
* Also know as the "Link-Belt" method in the USA.
* Fabrica de Aço Paulista S.A. (Faço), Manual de Transportadores Continuos, 1978, Brazil.
* Golka, et. al., Belt Conveyor Principles for Calculation and Design, 2007, Australia.
* Used today by … Google "Sitzwohl Pulley"

Shell - Sitzwohl

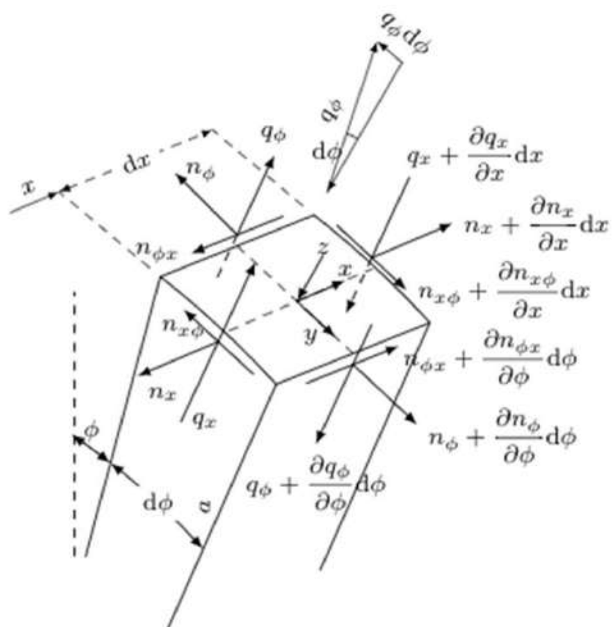
* Simple. Can be reduced to a table and a hand calculation.
* Treats cylinder as infinitely long – i.e. a loaded ring
* Only considers tangential bending stress in the shell.
* Also reference:
  * Roark, 1938 (1,2,34, editions) Table VIII, Case 15 and 25
  * Van den Broek, 1942, Chapter on Redundant Beams, Pipe Problems, Case II and Case VII.

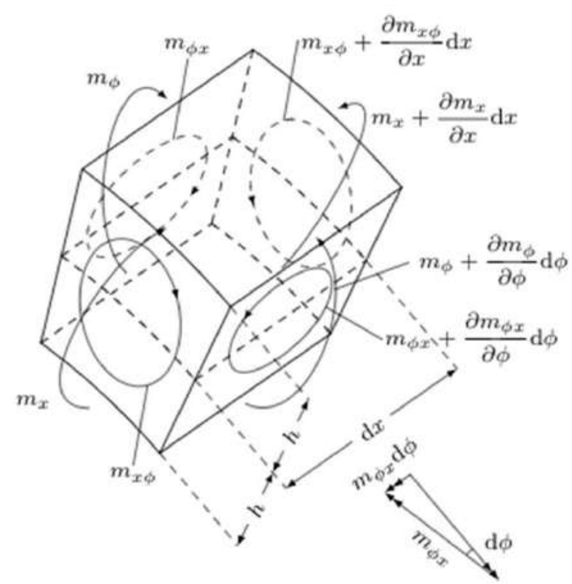
Shell Sitzwohl

* Short comings
  * Not very accurate.
  * Uniform loading, does not handle different T_1 and T_2, and shear force on the pulley.
  * Does not consider bending stress in the axial direction.
  * Does not consider any interaction with the end disk.
  * Does not accommodate internal stiffening disks.

Shell - Lange

* Lange, Hellmuth, “Untersuchungen zur Beanspruchung von Forderbandtrommeln”, “Technischen Hochschule Hannover”, 1963.





www.PulleyMaven.com

Shell - Lange

* "Moment Theory of Circular Cylindrical Shells"
* Complicated, intricate, advanced math.
* Helpful to read Girkmann, 1963 and Flugge, 1960 to understand the Lange Shell equations.
* After a lot of math, a bit of "luck", for a set of boundary conditions favorable for pulleys an exact solution is available.
* The boundary conditions are that at the end disk there is **no moment**, axial force, or radial deformation.

Shell Lange

* For 1963 this was very advanced numerical modeling work.
* Computers were in their infancy so a lot of focus on simplifying the results.
* Assumed that "critical" conveyor would have a wrap of 180 degrees so provided simplifications with this assumption. For the pulley shell smaller or larger wrap angles have higher stresses so this is not a good simplification to follow.
* Ringfeder locking assemblies were also just starting to be used. However, pressures were much lower, diameters much larger, and their impact on shell stresses not considered.
* Several companies use pulley models based on Lange – primarily in Europe and Australia.

Shell Lange

Advantages

* Considers the tangential, axial, and shear stresses in the shell.
* Good agreement with strain gauge measurements and FEA results in the center of the shell.
* With todays computers equations can be solved instantly. No need for some of the Lange and Schmoltzi simplifications.

Disadvantages

* Need a computer program to solve.
* Does not accommodate center disks.
* Load applied at the midplane on the shell. (Not shell surface, lagging surface, or belt carcass midplane)
* **Does not consider influence of the end disk and locking assembly.**

End Disk

"Simple Pulley Design"

* Good to explain the concept of "Rigid" versus "Flexible" end disk design.
* Good to explain the bending moment load sharing between the end disk and shaft.
* Not applicable today except possible for small, light duty pulleys.

Lange / Schmoltzi "Turbine" End Disk

* For large diameter, smaller hub, pulleys a reasonable choice.
* Using for small diameter, wide, high tension, high pressure locking assembly pulleys leads to a lot of uncertainty.